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ABNORMAL BEHAVIOR OF PERIODIC STRUCTURES

K. B. THAPA*, A. GUPTA, S. P. OJHA
Department of Physics, University Institute of Engineering & Technology
Chhatrapati Shahu Ji Maharaj University, Kanpur-208024(UP), INDIA

The dispersion characteristics of a one-dimensional periodic structure are analyzed on the basis of the exactly solvable Kronig–Penney model for the case of a periodic system consisting of an infinite number of layers arranged in the direction of propagation of radiation. An analytical solution is obtained for dispersion relation of the structure, and the behavior of phase and group velocities near the edges of the band structure is studied. The abnormal behavior of the periodic structure is qualitatively analyzed by the phase velocity and the group velocity with the use of the dispersion characteristics obtained by expanding the solution of the wave equation in terms of the harmonics of the lattice spacing.

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1. Introduction

Periodic dielectric structures characterized by a periodically varying refractive index with a period of the order of the wavelength of light and a large modulation depth are referred to as photonic crystals. As is known, a remarkable property of such structures is the presence of band gaps or stop bands (frequency ranges where the propagation of light waves is forbidden), which by analogy with the physics of solids are called photonic band gaps [1, 2]. This photonic band gap of the periodic structure or photonic crystal can act as a perfect mirror for electromagnetic wave with a frequency and can localize the wave modes if there are any defects in its structure. Such arrangement is commonly used in dielectric mirrors and filters. In other words the photonic crystals are materials patterned with a periodicity in dielectric constant, which can create a range of 'forbidden' frequencies. Photons with energies lying in the band gap cannot propagate through the medium. This provides the opportunity to shape and mould the flow of light for photonic information technology. These are artificial structures with high index contrast, the resultant photonic dispersion exhibits a band nature analogous to the electronic band structure in a solid, and the propagation of electromagnetic waves are forbidden in this PBG.

The one dimensional periodic system can exhibit three important phenomena: photonic band gap (PBG), localized mode and surface state. Due to easy fabrication point of view, the one dimensional periodic system is the simplest possible case for periodic structure which is also known as one dimensional photonic crystal (1-D PC). To understand the propagation of light through the 1-D PC, the principles of electromagnetism and symmetry have applied by several researchers [3, 4]. The simplest possible 1-D photonic crystal is shown in figure 1 which consist an alternating layer of materials with different dielectric constants like quarter wave stacks or dielectric mirrors. This arrangement is not a very new structure but the optical properties of such periodic structure (1-D PC) have been used widely in making reflectors and filters [3-5]. Using Kroning-Penney model and Bloch-Floquet theorem for binary periodic structure, the band structure of the periodic structure has been calculated. The dispersion relation of the electromagnetic wave modes of the 1-D PC is obtained by Khem et al. [6]. For this they have

*Corresponding author: khem.bhu@gmail.com

considered the wave functions in each medium to characterize the electromagnetic radiation waves and applied the boundary condition at each interface.

In contrast to the case of particles for which the velocity has a single meaning but for the waves have three different kinds of velocities: phase velocity, group velocity and energy velocity. These velocities are equal to each other in uniform materials with refractive indices which are real and independent of the frequency [6]. The phase velocity is defined as velocity of propagation of an equi-phase surface. This velocity has definite meaning, for e.g. the plane wave & spherical wave for which the equi-phase surface can be defined without ambiguity. In the periodic structure (PC) of dielectric materials, however, the equi-phase surfaces cannot be defined rigoursly. Since its eigen function is a superposition of plane waves. This means that the phase velocity cannot be define appropriately in the PC. On the other hand the group velocity which is a velocity of the propagation of a wave packet can be define as $V_g = \frac{\partial\omega}{\partial k(\omega)}$. The energy velocity is defined as the velocity of propagation of the electromagnetic energy. The propagation of electromagnetic energy by Poynting vector so that energy velocity is equal to ratio of time average Poynting vector to time average electromagnetic energy density [7, 8]. Inside the PC the phase velocity cannot be considered because equi-phase surface does not exist inside the periodic structure as discussed earlier so that the velocity inside the periodic structure may be considered group velocity or energy velocity. The group velocity is equal to energy velocity even though the dielectric constant is modulated periodically.

Using the group velocity concept inside the photonic crystals, Dowling et al. [9] have studied the photon group velocity near the band edges on 1-D photonic band gap structure. They observed that photon group velocity near the band edge approximately zero. This effect implied an exceedingly long optical path length in the structure. Such new effect has important application to vertical cavity surface emitting laser. Melnikov and Kozina [10] have studied the dispersion characteristics of 1-D photonic crystal on the basis of Kronig-Penney model for particular case of a periodic system consisting of an infinite number of insulating air (n=1.0) layer arrange in the direction of propagation of radiation. They have shown that no gain in PBG is present because of interference quenching of the wave inside the structure. The study reveals the group velocity inside the 1-D PC has reason to produce the negative refraction in finite 1-D photonic crystal.

In this paper, the dispersion characteristics of a one-dimensional periodic structure are analyzed on the basis of the exactly solvable Kronig–Penney model for the case of a periodic system consisting of an infinite number of layers arranged in the direction of propagation of radiation. The binary periodic structure contains n_1 and n_2 refractive indices of the thicknesses d_1 and d_2 respectively and thickness of the unit cell is $d=d_1+d_2$. An analytical solution is obtained for the dispersion relation, and the behavior of phase and group velocities near the edges of the band structure is studied. The abnormal behavior of the periodic structure is qualitatively analyzed by the phase velocity and the group velocity with the use of the dispersion characteristics.

2. Theory

It is well known that when electrons move through a periodic potential lattice, the allowed and forbidden energy bands are obtained. The same idea may be applicable to the case of optical radiation if the electrons are replaced by optical waves (photons) and the periodic potential is replaced by a periodic refractive index pattern [9, 10, 11]. One expects allowed and forbidden bands of frequencies instead of energies is obtained for certain electromagnetic wave. By choosing a linearly periodic refractive index profile in the periodic dielectric material one obtains a given set of wavelength ranges that are allowed or forbidden to pass through the periodic dielectric material. Selecting a particular x-axis through the material, we shall assume a periodic step function for the index of the form;

$$n(x) = \begin{Bmatrix} n_1, & 0 \le x \le d_1 \\ n_2, & -d_2 \le x \le 0; \end{Bmatrix} \tag{1}$$

where n(x)=n(x+md) and m is the translation factor, which takes the values m=0, ±1, ±2, ±3,..., and $d=d_1+d_2$ is the period of the lattice with d_1 and d_2 is being the width of the two regions having refractive indices n_1 and n_2 respectively. The refractive indices profile of the materials in the form of rectangular symmetry is shown in the figure 1.

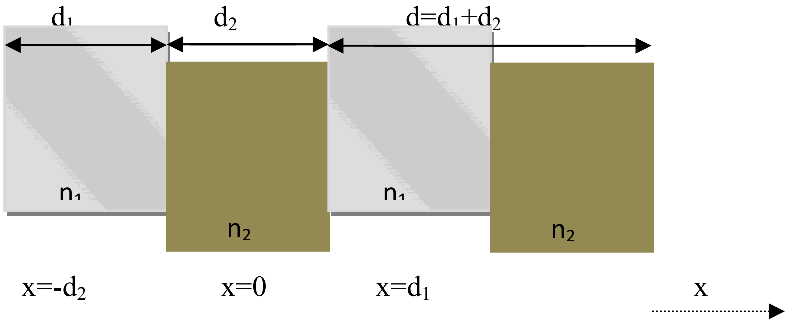

Fig. 1. Periodic refractive index profile of dielectric materials.

If θ is the angle of incident on this periodic structure the one dimensional wave equation for the spatial part of the electromagnetic eigen mode ψ_k (x) is given as,

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n^2(x)\cos^2\theta.\omega^2{}_k}{c^2}\psi_k(x) = 0, \tag{2}$$

where n(x) is given by equation (1). Therefore, equation (2) for wave equation for two media may be written as,

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n_1^2(x)\cos^2\theta_1.\omega^2{}_k}{c^2}\psi_k(x) = 0; \qquad 0 \le x \le d_1 \tag{3a}$$

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n_2^2(x)\cos^2\theta_2.\omega^2{}_k}{c^2}\psi_k(x) = 0; \qquad -d_2 \le x \le 0 \tag{3b}$$

where θ_1 and θ_2 are ray angle in the layer of refractive index n_1 and n_2 respectively. The periodic nature of the problem allows the application of Bloch's theorem [2] which solution can be written as $\psi(k) = u_k(x)\, e^{ikx}$ where K is known as Bloch wave number and $u_k(x)$ is the value of the eigen function. Thus using Bloch's theorem, these equations (3a) and (3b) can be written as;

$$\frac{d^2u_1}{dx^2} + 2iK\frac{du_1}{d_x} + \left(\alpha^2 - K^2\right)u_1 = 0; \qquad 0 \le x \le d_1 \tag{4a}$$

$$\frac{d^2u_2}{dx^2} + 2iK\frac{du_2}{d_x} + \left(\beta^2 - K^2\right)u_2 = 0; \qquad -d_2 \le x \le 0 \tag{4b}$$

where $\alpha = \left(\frac{n_1\omega}{c}\cos\theta_1\right), \beta = \left(\frac{n_2 w}{c}\cos 0_2\right), \theta_1 = \cos^{-1}\left[1 - \frac{\sin^2\theta}{n_1}\right]^{1/2},$

$\theta 2 = \cos^{-1}\left[1 - \frac{\sin^2\theta}{n_2}\right]^{1/2}$ and u_1 represents the value of $u_k(x)$ in the interval $(0,d_1)$ and u_2 in the interval $(-d_2,0)$ respectively. The solution of differential equations (4a) and (4b) can be written as,

$$u_1 = Ae^{i(\alpha-K)x} + Be^{-i(\alpha+K)x} \tag{5a}$$

$$u_2 = Ce^{i(\beta-K)x} + De^{-i(\beta+K)x} \tag{5b}$$

Now applying the boundary conditions as given below:

$$u_1(x)\big|_{x=0} = u_2(x)\big|_{x=0} \tag{6a}$$

$$u'_1(x)\big|_{x=0} = u'_2(x)\big|_{x=0} \tag{6b}$$

$$u_1(x)\big|_{x=d1} = u_2(x)\big|_{x=d2} \tag{6c}$$

$$u'_1(x)\big|_{x=d1} = u'_2(x)\big|_{x=d2} \tag{6d}$$

We get four equations having four unknown constants. To obtain a nontrivial solution for the equations, the determinant of the coefficients of the unknown constants must be zero, which is given as,

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix} = 0$$

where $A_{11}=A_{12}=A_{13}=A_{14}=1$; $A_{21}=i\,(\alpha\text{-}K)$, $A_{22}=-i\,(\alpha+K)$, $A_{23}=i\,(\beta\text{-}K)$, $A_{24}=-i\,(\beta+K)$; $A_{31}=e^{id_1(\alpha-K)}$, $A_{32}=e^{-id_1(\alpha+K)}$, $A_{33}=e^{-id_2(\beta-K)}$, $A_{34}=e^{id_2(\beta+K)}$; $A_{41}=i\,(\alpha\text{-}K)\,e^{id_1(\alpha-K)}$ $A_{42}=e\text{-}i(\alpha+K)e^{-id_1}(\alpha+K)$, $A_{43}=i\,(\beta\text{-}K)\,e^{id_2(\beta-K)},$ $A_{44}=i(\beta+K)e^{-id_2}(\beta+K)$.

On solving above equation, we obtain,

$$\cos(K.d) = \cos(\alpha d_1).\cos(\beta d_2) - \frac{1}{2}\left(\frac{n_1.\cos\theta_1}{n_2.\cos\theta_2} + \frac{n_2.\cos\theta_2}{n_1.\cos\theta_1}\right).\sin(\alpha d_1).\sin(\beta d_2) \tag{7}$$

Now, abbreviating the L.H.S. as L_ω equation (7) may be written as,

$$L(\omega) = \mathrm{Cos}\,(K.d) \tag{8}$$

Using equation (8), we may be written the dispersion relation as,

$$K(\omega) = \frac{1}{d}\cos^{-1}(L(\omega)) \tag{9}$$

For calculating the phase (V_p) and group (V_g) velocities of the considered structure as suggested by Dowling et al. [9], we solved the equation (9) [11]. We obtain,

$$v_p = \frac{\omega}{K(\omega)} \tag{10}$$

and

$$v_g = \frac{1}{\dfrac{\partial K(\omega)}{\partial \omega}} \tag{11}$$

3. Results and discussion

We have considered a periodic structure containing n_1 and n_2 refractive indices with thicknesses d_1 and d_2 respectively and thickness of the unit cell $d=d_1+d_2$. We have obtained dispersion relation by using Kronig Penney model and Bloch theorem. To study the abnormal behavior of periodic structure, we have calculated phase velocity and group velocity of structure at the band edges for the following compositions of refractive indices: (i) $n_1=2.35$, $n_2=1.5$ & $n_1=2.35$, $n_2=4.2$ and (ii) $n_1=1.0$, $n_2=1.5$ & $n_1=1.0$, $n_2=4.2$ with $n_1d_1=n_2d_2=\lambda_0/4$ for normalized wavelength.

Figure 2 represents the cosine wave and band structure versus normalized frequency for periodic structure having n_1=2.35, n_2=1.5 (solid line) & n_1=2.35, n_2=4.2 (dot line) of the thicknesses $d_1=\lambda_0/4n_1$ & $d_2=\lambda_0/4n_2$. The phase velocity and group velocity corresponding dispersion relation versus normalized frequency for same structure having n_1=2.35, n_2=1.5 (solid line) & n_1=2.35, n_2=4.2 (dot line) of the thicknesses $d_1=\lambda_0/4n_1$ & $d_2=\lambda_0/4n_2$ is depicted in the figure 3. The cosine wave and band structure of the considered structure is increased when the refractive index contrast of the structure increases. The solid line of the cosine wave and band structure show the periodic structure for the contrast of the refractive index ($\Delta n=n_1 \sim n_2$) is about 0.85. Similarly the dot line of the cosine wave and band structure show the periodic structure for the contrast of the refractive index ($\Delta n=n_1 \sim n_2$) is about 1.85. For large refractive index contrast, the cosine wave and band structure is found large. The forbidden band gaps of the considered structure are approximately 2.0 and 2.4 of the normalized frequencies for Δn=0.85 and Δn=1.85 respectively.

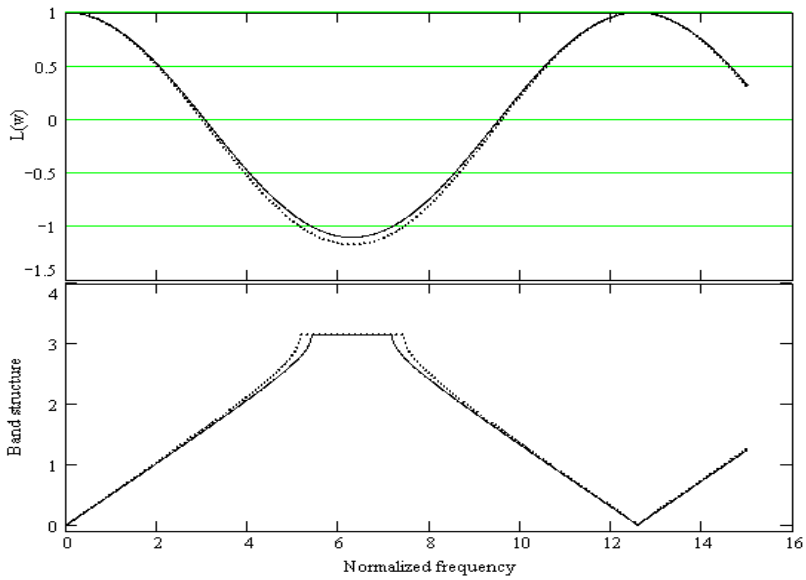

Fig. 2. Cosine wave and dispersion relation for n_1=2.35, n_2=1.5 (solid line) & n_1=2.35, n_2=4.2 (dot line)

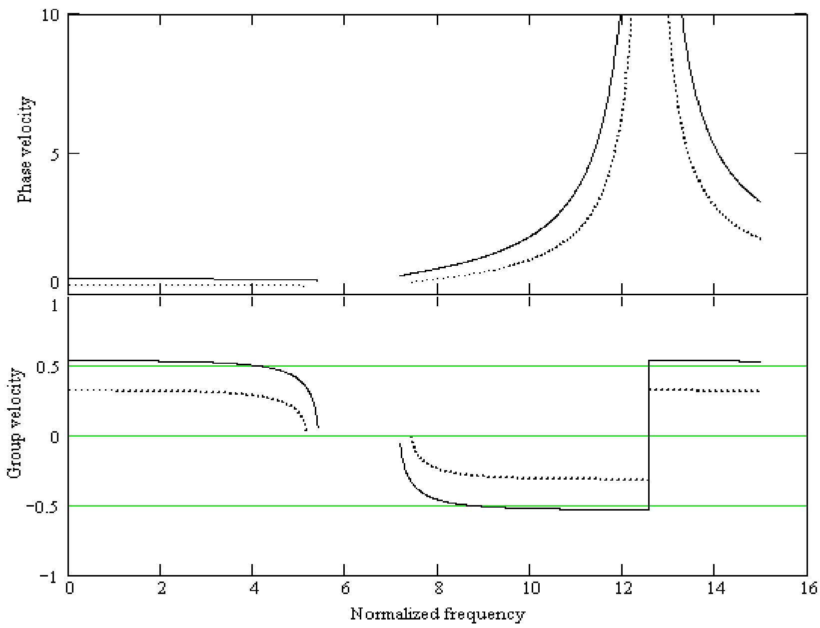

Fig. 3. Phase velocity and group velocity for n_1=2.35, n_2=1.5 (solid line) & n_1=2.35, n_2=4.2 (dot line)

The phase velocity and group velocity of the structure are calculated using band structure or dispersion relation of the structure. The phase and group velocities are found large for the low contrast of the refractive index compare to the high contrast of the refractive index. The forbidden band gap of the structure lies at the 5-7 of normalized frequency and other frequency range are allowed band gap for the same structure. We are theoretically interested at the band edges of the forbidden normalized frequency 5-7 and allowed normalized frequency 12.5 only. The phase velocity at the band edges for the forbidden band gap is slightly changed but discontinuous for forbidden band gap and becomes very high at 12.5 frequency point due to abnormal behavior of the periodic structure. The group velocity of the structure is become zero near the band edges and discontinuous for the forbidden band gap. The group velocity at the 12.5 frequency is exactly zero for both sides. The group velocity is found negative between the 7.2-12.5 frequency ranges due to the local curvature of the equifrequency contour (EFC). The group velocity in the EFC is directed away from the normal on the same side as the incident beam. Moreover, if the EFCs are approximately circular, and group velocity is directed radially inwards, then the PC displays negative refraction for all incident angles.

Similarly figure 4 represents the cosine wave and band structure versus normalized frequency for periodic structure with $n_1=1.0$, $n_2=1.5$ (solid line) & $n_1=1.0$, $n_2=4.2$ (dot line) of the thicknesses $d_1=\lambda_0/4n_1$ & $d_2=\lambda_0/4n_2$. The phase velocity and group velocity corresponding dispersion relation versus normalized frequency for structure with $n_1=1.0$, $n_2=1.5$ (solid line) & $n_1=1.0$, $n_2=4.2$ (dot line) of the thicknesses $d_1=\lambda_0/4n_1$ & $d_2=\lambda_0/4n_2$ is depicted in figure 5. The cosine wave and band structure of the considered structure is also increased when the refractive index contrast of the structure increases. The solid line of the cosine wave and band structure show the periodic structure for the contrast of the refractive index ($\Delta n=n_1\sim n_2$) is about 1.5. Similarly the dot line of the cosine wave and band structure show the periodic structure for the contrast of the refractive index ($\Delta n=n_1\sim n_2$) is about 4.2. For large refractive index contrast, the cosine wave and band structure is also found large. The forbidden band gaps of the considered structure are approximately 2.1 and 6.4 normalized frequencies for $\Delta n=1.5$ and $\Delta n=4.2$ respectively. The phase and group velocities are also found large for the low contrast of the refractive index compare to the high contrast of the refractive index. The phase velocity at the band edges for the forbidden band gap is also changed but discontinuous for forbidden band gap due to anomalously change of the refractive index of the structure. The phase velocity becomes also very high at 12.5 frequency point due to abnormal behavior of the periodic structure. The group velocity of the structure is also become zero near the band edges and discontinuous for the forbidden band gap. The group velocity at the 12.5 frequency is exactly zero for both sides. The phase velocity and group velocity of the structure are found large compare to the structure with low refractive index contrast. From above study, we conclude that the large contrast of the refractive index has large forbidden band gap but the low contrast of the refractive index has large group velocity. Due to abnormal behavior of the periodic structure, the velocities at the forbidden band edges becomes zero and exactly zero at the allowed band edge. However the group velocity is found to be large in those periodic structures which exists large forbidden band gap. The group velocity of the structure is found negative between the high frequency forbidden band edge and the low frequency allowed band edge due to the local curvature of the equi-frequency contour (EFC).

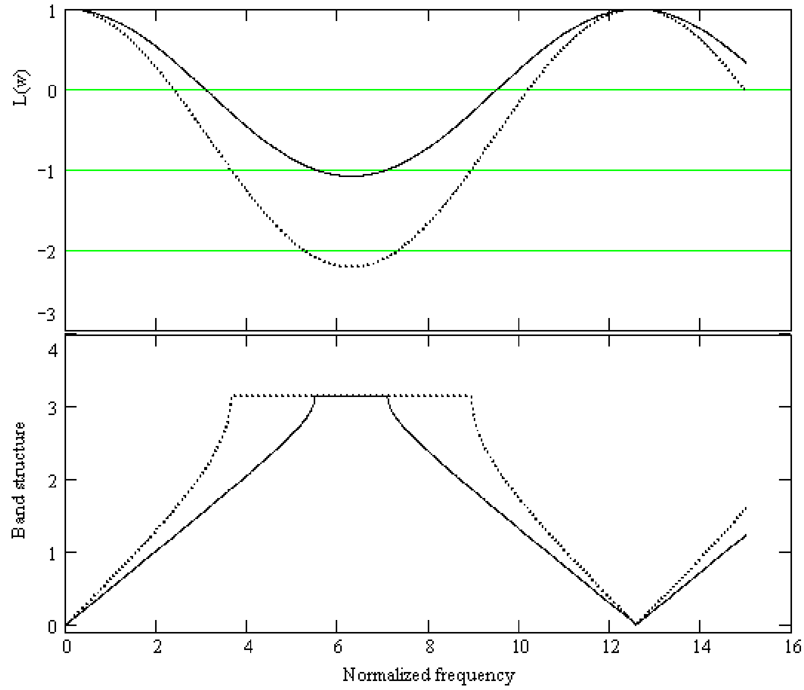

Fig. 4. Cosine wave and dispersion relation for n_1=1.0, n_2=1.5 (solid line) & n_1=1.0, n_2=4.2 (dot line)

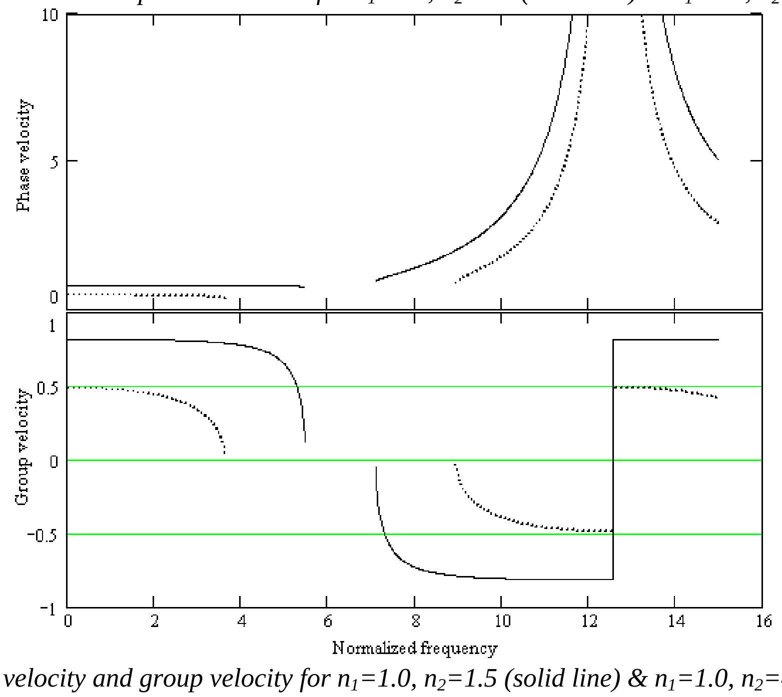

Fig. 5: Phase velocity and group velocity for n_1=1.0, n_2=1.5 (solid line) & n_1=1.0, n_2=4.2 (dot line).

4. Conclusions

The dispersion characteristics of a one-dimensional periodic structure have analyzed on the basis of the exactly solvable Kronig–Penney model for the case of a periodic system consisting of an infinite number of layers arranged in the direction of propagation of radiation. An analytical solution is obtained for the dispersion relation, and the behavior of phase and group velocities near the edges of the band structure is studied. Due to abnormal behavior of the periodic structure, the velocities at the forbidden band edges becomes zero and exactly zero at the allowed band edge. The group velocity of the structure is found negative between the high frequency forbidden band edge and the low frequency allowed band edge due to the local curvature of the equi-frequency contour (EFC). We have also found that the large contrast of the refractive index has large forbidden band gap and the low contrast of the refractive index has large group velocity due to the abnormal behavior of the periodic structure.

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